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ALLOCATION TABLE METHOD TO SOLVE SPECIAL CASES OF TRANSPORTATION PROBLEM

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ABSTRACT

To find an Initial Basic Feasible Solution (IBFS) is the prime requirement to obtain the Optimal Solution for the Transportation Problem. The purpose of this paper is to find the IBFS for prohibited and maximization transportation problem. Comparatively, applying Allocation Table Method (ATM) in the proposed method obtains the best IBFS to the transportation problem and performs faster than the existing methods with a minimal computation time and less complexity. Numerical examples are presented to verify the proposed approach.

Keywords: Allocation Table Method (ATM), Initial Basic Feasible Solution (IBFS), Maximization Transportation Problem, Prohibited Transportation Routes, Transportation Model, Vogel's Approximation Method.

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1. Introduction

The most important and successful applications in the optimization refers to Transportation Problem (TP), that is a special class of the linear programming (LP) in the operations research (OR). This is a special kind of the network optimization problems in which goods are transported from a set of sources to a set of destinations subject to the demand and supply of the source and destination respectively, such that the total cost of transportation is minimized. The basic transportation problem was originally developed by Hitchcock in 1941[3]. Efficient methods for finding solution were developed, primarily by Dantzig in 1951[2] and then by Charnes, Cooper and Henberson in 1953[1]. Basically, the solution procedure for the transportation problem consists of the following phases:

Phase 1: Mathematical formulation of the transportation problem.

Phase 2: Finding an IBFS.

Phase 3: Optimize the IBFS which is obtained in Phase 2.

In this paper, Phase 2 has been focused in order to obtain a better IBFS for the prohibited transportation routes and maximization transportation problem by using Allocation Table Method (ATM) [5].

2. Preliminaries

2.1. Transportation Model [5]:

In a transportation problem, we focus on the original points. These points may represent factories to produce items and to supply a required quantity of the products to a certain number of destinations. This process must be done successfully in such a way as to maximize the profit or minimize the cost transportation. Therefore, the places of production and supply are collected as the original points and the destinations respectively. Sometimes the original and destinations points are also termed as sources and sinks. However, to illustrate a typical transportation model, suppose that m factories supply certain items to n warehouses. As well as, let factory i ($i = 1, 2, \dots, m$) produce a_i units and the warehouse j ($j = 1, 2, \dots, n$) requires b_j units. Furthermore, suppose the cost of transportation from factory i to warehouse j is c_{ij} . The decision variables x_{ij} is being the transported amount from factory i to warehouse j . Typically, our objective is to find the transportation pattern which will minimize the total of the transportation cost (Table 1).

Table 1: The model of a transportation problem

Origins (Factories)	Destinations (Warehouses)				Available
	1	2	...	n	
1	c_{11}	c_{12}	...	c_{1n}	a_1
2	c_{21}	c_{22}	...	c_{2n}	a_2
...
M	c_{m1}	c_{m2}	...	c_{mn}	a_m
Required	b_1	b_2	...	b_n	

2.2. Prohibited Transportation Routes [4]:

The situation may arise such as road hazards (snow, flood, etc.), traffic regulations, etc., when it is not possible to transport goods from certain sources to certain

destinations. Such type of problems can be handled by assigning a very large cost say M to that route (or cell).

2.3. Maximization Transportation Problem [4]:

In general, transportation method is used for minimization problems. However it may also be used to solve problems in which the objective is to maximize when we consider the unit profit (or payoff) p_{ij} instead of unit cost c_{ij} associated with each route (i, j) . The solution procedure for solving such problems is summarized below:

Step 1: Convert the given problem into that of minimization by replacing each element of the transportation table by its difference from the maximum element of the table.

Step 2: Find an initial feasible solution using any of the methods.

Step 3: Use MODI method for finding the optimum solution.

3. Algorithms

There are several algorithms for solving transportation problems which are based on differences of special linear programming methods. Basically, those methods are different in terms of the quality for the produced starting solution and the best starting solution that yields smaller objective value. In this study, we used the Vogel's Approximation Method (VAM), since it generally produces better starting solutions than other solving methods: as well as we have used the ATM solution steps [5].

3.1. Vogel's Approximation Method (VAM) [4]:

The Vogel's Approximation Method takes into account not only the least cost c_{ij} but also the costs that exceed c_{ij} . The steps of the method are given below:

Step 1: For each row of the transportation table identify the smallest and the next-to-smallest costs. Determine the difference between them for each row. Display them alongside the transportation table by enclosing them in parenthesis against the respective rows. Similarly, compute the differences for each column.

Step 2: Identify the row or column with the largest difference among all the rows and columns. If a tie occurs, use any arbitrary tie-breaking choice. Let the greatest difference correspond to i^{th} row and let c_{ij} be the smallest cost in the i^{th} row. Allocate the maximum feasible amount $x_{ij} = \min(a_i, b_j)$ in the $(a_i, b_j)^{th}$ cell and cross off either the i^{th} row or j^{th} column in the usual manner.

Step 3: Recompute the column and row difference for the reduced transportation table and go to step 2. Repeat the procedure until all the rim requirements are satisfied.

3.2. Allocation Table Method (ATM) [5]:

In this proposed approach, an Allocation Table (AT) is formed to find the solution for the transportation problem. That's why this method is named as Allocation Table Method (ATM) and the method is illustrated below:

Step 1: Construct a Transportation Table (TT) from the given transportation problem.

Step 2: Ensure whether the TP is balanced or not, if not make it balanced.

Step 3: Select Minimum Odd Cost (MOC) from all the cost cells of TT. If there is no odd cost in the cost cells of the TT, keep on dividing all the cells by 2 till obtaining at least an odd value in the cost cells.

Step 4: Form a new table which is to be known as allocation table (AT) by keeping the MOC in the respective cost cell/cells as it was/were, and subtract selected MOC only from each of the odd cost valued cells of the TT. Now all the cell values are to be called as Allocation Cell Value (AVC) in AT.

Step 5: At first, start allocation from minimum of supply/demand. Allocate this minimum of supply/demand in the place of odd valued ACVs at first in the AT formed in Step 4. If demand is satisfied, delete the column. If it is supply, delete the row.

Step 6: Now identify the minimum AVC and allocate minimum of supply/demand at the place of selected ACV in the AT. In case of same ACVs, select the ACV where minimum allocation can be made. Again, in case of same allocation in the AVCs, choose the minimum cost cell which corresponds to the cost cells of TT formed in Step 1 (i.e. this minimum cost cell is to be found out from the TT which is constructed in Step 1). Again, if the cost cells and the allocations are equal, in such case choose the nearer cell to the minimum of demand/supply which is to be allowed. Now if demand is satisfied delete the column and if it is supply delete the row.

Step 7: Repeat Step 6 until the demand and supply are exhausted.

Step 8: Now transfer this allocation to the original TT.

Step 9: Finally calculate the total transportation cost of the TT. This calculation is the sum of the product of cost corresponding allocated value of the TT.

4. Numerical Problems

4.1 Special Case 1: (Prohibited Transportation Routes)

4.1.1 Problem 1 [4]:

Consider the following transportation problem:

Table 2: Data of the Problem 1

Factory	Godowns						Stock Available
	1	2	3	4	5	6	
A	7	5	7	7	5	3	60
B	9	11	6	11	-	5	20
C	11	10	6	2	2	8	90
D	9	10	9	6	9	12	50
Demand	60	20	40	20	40	40	220

Solution of Problem 1:

4.1.1.(a) Initial Basic Feasible Solution according to the VAM is shown in the Table 3.

Table 3: Initial Basic Feasible Solution according to VAM with allocation

Factory	Godowns						Stock Available
	1	2	3	4	5	6	
A		20			E	40	60
	7	5	7	7	5	3	
B	10		10				20
	9	11	6	11	M	5	
C			30	20	40		90
	11	10	6	2	2	8	
D	50						50
	9	10	9	6	9	12	
Demand	60	20	40	20	40	40	220

➤ The total transportation cost is $(20 * 5 + \epsilon * 5 + 40 * 3 + 10 * 9 + 10 * 6 + 30 * 6 + 20 * 2 + 40 * 2 + 50 * 9 =) 1120$.

4.1.1.(b) Allocation of various cells in the allocation table of problem 1 is shown in Table 4.

Table 4: Allocation of various cells are in the allocation table.

Factory	Godowns						Stock Available
	1	2	3	4	5	6	
A	4	20 2	4	4	2	40 3	60
B	20 6	8	6	8	M	2	20
C	8	10	30 6	20 2	40 2	8	90
D	40 6	10	10 6	6	3	12	50
Demand	60	20	40	20	40	40	220

Illustration:

- According to Step 2: It is found that the given problem is balanced because of the sum of stocks available = sum of the demands = 220.
- As per Step 3, minimum odd cost is 3 (1, 6) among all the cost cells of the Transportation Table 2.
- Allocation Table 4, is formed according to step 4, where minimum odd cost is in cell (1, 6) remains same, but this odd cost is subtracted from all other odd valued cost cells of the transportation Table 2. Like in cost cell (1, 1) it is 7 in transportation Table 2, but in allocation table this cell value is $(7 - 3) = 4$.
- According to Step 5, minimum stock/demand is 40 that is allocated in cell (1, 5). After allocating this value it is found that the demand is satisfied. For which column 6 is to be exhausted.
- After Step 5, only the cells of 1, 2, 3, 4 and 5 columns are to be considered. Where 2 is the lowest cell value in the cells (1, 2), (1, 5), (3, 4) and (3, 5). Among these four cells 20 is the lowest allocation that can be made in cell (1, 2). Now row A is crossed out after allocating this amount.
- Again, it is found 2 is the lowest cell value in the remaining cells which appears in the cells (3, 4) and (3, 5). Among these two cells, minimum allocation 20 is made in the cell (3, 4). So, the cells of column 4 are not to be considered for further calculation.

- After doing the above allocation, it is found 2 is again the minimum cell value that appears in the cell (3, 5). So, the cell is allocated with minimum value 30 such that row 3 is exhausted.
- Finally complete the allocation by allotting 10, 20 and 40 in the cells (4, 3), (2, 1) and (4, 1) respectively. All these allocations are made according to Step 6 and Step 7 of the Allocation Table Method.
- Now according to Step 8, all these allocations are transferred to the Transportation Table 2, which is shown in the Final Allocation Table 5. In this table it is found that the solution is non-degenerate which represents the IBFS according to Allocation Table Method.

Table 5: Initial Basic Feasible Solution according to ATM

Factory	Godowns						Stock Available
	1	2	3	4	5	6	
A	7	20 5	7	7	E 5	40 3	60
B	20 9	11	6	11	M	5	20
C	11	10	30 6	20 2	40 2	8	90
D	40 9	10	10 9	6	9	12	50
Demand	60	20	40	20	40	40	220

➤ Finally, according to Step 9, the total transportation cost is $(20 * 5 + \epsilon * 5 + 40 * 3 + 20 * 9 + 30 * 6 + 20 * 2 + 40 * 2 + 40 * 9 + 10 * 9 =)$ **1150**.

4.1.2. Problem 2 [7]: (Numerical Example without illustrations)

Consider the following transportation problem:

Table 6: Data of the Problem 2

Factory	A	B	C	Supply
1	25	21	19	120
2	15	7	-	150
3	10	12	16	80
Demand	150	125	75	350

Solution of Problem 2:

4.1.2.(a) Initial Basic Feasible Solution according to the VAM is shown in the Table 3.

Table 7: Initial Basic Feasible Solution according to VAM with allocation

Factory	A	B	C	Supply
1	45 25	21	75 19	120
2	25 15	125 7	M	150
3	80 10	12	16	80
Demand	150	125	75	350

➤ The total transportation cost is $(45 * 25 + 75 * 19 + 25 * 15 + 125 * 7 + 80 * 10 =)$ **4600**.

4.1.2.(b) Initial Basic Feasible Solution according to ATM is shown in the Table 8.

Table 8: Initial Basic Feasible Solution according to ATM

Factory	A	B	C	Supply
1	120 25	21	19	120
2	25 15	125 7	M	150
3	5 10	12	75 16	80
Demand	150	125	75	350

➤ The total transportation cost is $(120 * 25 + 25 * 15 + 125 * 7 + 5 * 10 + 75 * 16 =)$ **5500**.

4.2. Special Case 2: (Maximization Transportation Problem)

4.2.1 Problem 1 [6]:

Consider the following transportation problem:

Table 9: Data of the Problem 1

Warehouse	A	B	C	D	Demand
1	6	4	10	15	100
2	4	3	8	11	200
3	10	7	12	10	120
4	1	7	13	9	80
5	9	9	7	8	70
Capacity	150	250	100	70	570

Solution of Problem 1:

4.2.1.(a) Step 1: Converting the given maximization problem to minimization one by subtracting all the element of the table from the maximum element 15. The minimization transportation problem will be

Table 10: Transportation Table

Warehouse	A	B	C	D	Demand
1	9	11	5	0	100
2	11	12	7	4	200
3	5	8	3	5	120
4	14	8	2	6	80
5	6	6	8	7	70
Capacity	150	250	100	70	570

Step 2: Initial Basic Feasible Solution according to the VAM is shown in the Table 11.

Table 11: Initial Basic Feasible Solution according to VAM with allocation

Warehouse	A	B	C	D	Demand
1	10 6	4	20 10	70 15	100
2	20 4	180 3	8	11	200
3	120 10	7	12	10	120
4	1	7	80 13	9	80
5	9	70 9	7	8	70
Capacity	150	250	100	70	570

- The total transportation cost is $(10 * 6 + 20 * 4 + 120 * 10 + 180 * 3 + 70 * 9 + 20 * 10 + 80 * 13 + 70 * 15 =)$ **4800**.

4.2.1.(b) Step 1: Converting the given maximization problem to minimization one by subtracting all the element of the table from the maximum element 15. The minimization transportation problem will be

Table 12: Transportation Table

Warehouse	A	B	C	D	Demand
1	9	11	5	0	100
2	11	12	7	4	200
3	5	8	3	5	120
4	14	8	2	6	80
5	6	6	8	7	70
Capacity	150	250	100	70	570

Step 2: Allocation of various cells in the allocation table of problem 1 is shown in Table 13.

Table 13: Allocation of various cells are in the allocation table.

Warehouse	A	B	C	D	Demand
1	30 6	8	2	70 0	100
2	30 8	170 12	4	4	200
3	20 2	8	100 3	2	120
4	14	80 8	2	6	80
5	70 6	6	8	4	70
Capacity	150	250	100	70	570

Illustration:

- According to Step 2: It is found that the given problem is balanced because of the sum of capacities = sum of the demands = 570.
- As per Step 3, minimum odd cost is 3 (3, 3) among all the cost cells of the Transportation Table 12.
- Allocation Table 13, is formed according to step 4, where minimum odd cost is in cell (3, 3) remains same, but this odd cost is subtracted from all other odd valued cost cells of the transportation Table 12. Like in cost cell (1, 1) it is 9 in transportation Table 12, but in allocation table this cell value is $(9 - 3) = 6$.
- According to Step 5, minimum stock/demand is 100 that is allocated in cell (3, 3). After allocating this value it is found that the capacity is satisfied. For which column C is to be exhausted.
- After Step 5, only the cells column 1, 2 and 4 are to be considered. Now the minimum element is 0 (1, 4) So, the cell is allocated with minimum value 70 such that column 4 is exhausted.
- Next lowest cell value is 2 in the cell (3, 1) in which 20 is the lowest allocation that can be made in cell (3, 1). Now row 3 is crossed out after allocating this amount.
- After doing the above allocation, it is found 6 is the minimum cell value that appears in the cells (1, 1), (5, 1) and (5, 2) lowest allocation. Among these three cells 30 is the lowest allocation. So, the cell is allocated with minimum value 30 such that row 1 is exhausted.

- Finally complete the allocation by allotting 70, 30, 80 and 170 in the cells (5, 1), (2, 1), (4, 2) and (2, 2) respectively. All these allocations are made according to Step 6 and Step 7 of the Allocation Table Method.
- Now according to Step 8, all these allocations are transferred to the Transportation Table 8, which is shown in the Final Allocation Table 14. In this table it is found that the number of basic cells is 8 ($= 5 + 4 - 1$) which represents the IBFS according to Allocation Table Method.

Table 14: Initial Basic Feasible Solution according to ATM

Warehouse	A	B	C	D	Demand
1	30 6	4	10	70 15	100
2	30 4	170 3	8	11	200
3	20 10	7	100 12	10	120
4	1	80 7	13	9	80
5	70				70
Capacity	150	250	100	70	570

- Finally, according to Step 9, the total transportation cost is $(30 * 6 + 30 * 4 + 20 * 10 + 70 * 9 + 170 * 3 + 80 * 7 + 100 * 12 + 70 * 15 =)$ **4450**.

4.2.2 Problem 2 [6]: (Numerical Example without illustrations)

Surya Roshni Ltd has three factories: X, Y and Z. It supplies goods to four dealers spread all over the country. The production capacities of these factories are 200, 500 and 300 per month respectively.

Table 15: Data of the Problem 2

Factory	Dealer				Capacity
	A	B	C	D	
X	12	18	6	25	200
Y	8	7	10	18	500
Z	14	3	11	20	300
Demand	180	320	100	400	1000

Solution of Problem 2:

4.2.2.(a) Finally, Initial Basic Feasible Solution according to the VAM is shown in the Table 16.

Table 16: Initial Basic Feasible Solution according to VAM with allocation

Factory	A	B	C	D	Capacity
X	12	200 18	6	25	200
Y	8	100 7	10	400 18	500
Z	180 14	20 3	100 11	20	300
Demand	180	320	100	400	1000

- The total transportation cost is $(180 * 14 + 200 * 18 + 100 * 7 + 20 * 3 + 100 * 11 + 400 * 18 =)$ **15180**.

4.2.2.(b) Finally, Initial Basic Feasible Solution according to the ATM is shown in the Table 17.

Table 17: Initial Basic Feasible Solution according to ATM

Factory	A	B	C	D	Capacity
X	12	100 18	6	100 25	200
Y	180 8	220 7	100 10	18	500
Z	14	3	11	300 20	300
Demand	180	320	100	400	1000

- The total transportation cost is $(180 * 8 + 100 * 18 + 220 * 7 + 100 * 10 + 100 * 25 + 300 * 20 =)$ **14280**.

5. Results

After obtaining an initial basic feasible solution for special cases by “Allocation Table Method (ATM)”, the obtained result is compared with the results obtained by VAM. ATM works to obtain the optimal solution or closest to optimal solution as it has a better starting solution for the maximization transportation problem. Whereas in prohibited transportation routes, VAM has a better starting solution with minimum transportation cost.

Therefore, ATM is best suited for maximization transportation problem and this method fails for prohibited transportation routes.

Conclusion

In this study, we have used ATM for finding an IBFS for special cases. Efficiency of allocation table method has also been tested by solving cost minimizing transportation problems and it yields comparatively a better result than VAM. Finally, ATM will reduce the complexity with a simple and provides a remarkable IBFS by ensuring minimum transportation cost only for maximization transportation problem.

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